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OLIGARCHY FOR SOCIAL CHOICE CORRESPONDENCES AND STRATEGY-PROOFNESS

ABSTRACT. We study the existence of a group of individuals which has some decisive power for social choice correspondences that satisfy a monotonicity property which we call *modified monotonicity*. And we examine the relation between modified monotonicity and strategy-proofness of social choice correspondences according to the definition by Duggan and Schwartz (2000). We will show mainly the following two results. (1) Modified monotonicity implies the existence of an oligarchy. An oligarchy is a group of individuals such that it has some decisive power (*semi-decisiveness*), and at least one of the most preferred alternatives of every its member is always chosen by any social choice correspondence. (2) Strategy-proofness of social choice correspondences is equivalent to modified monotonicity.

KEY WORDS: modified monotonicity, oligarchy, social choice correspondences, strategy-proofness, weak dictator.

1. INTRODUCTION

The Gibbard–Satterthwaite theorem (Gibbard (1973) and Satterthwaite (1975)) states that there is no social choice function which satisfies the conditions of unrestricted domain, strategy-proofness and non-dictatorship. They considered only resolute (single-valued) social choice functions which choose only one of the alternatives. Recently there are several works which studied the properties of strategy-proof social choice correspondences such as Duggan and Schwartz (2000), Ching and Zhou (2002) and Barbera, Dutta and Sen (2001)[1]. The definition of strategy-proofness by Duggan and Schwartz (2000) is weaker than the definition by Ching and Zhou (2002) and Barbera, Dutta and Sen (2001). According to the definition by Duggan and Schwartz (2000), if misrepresenting of the preference by an individual increases his expected utility evaluated by *some* utility function which represents his preference for *all* assignments of probabilities to alternatives, a social choice correspondence is manipulable. On the other hand, according to

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the definition by Ching and Zhou (2002) and Barbera, Dutta and Sen (2001), if misrepresenting of the preference by an individual increases his expected utility evaluated by *some* utility function which represents his preference for *some* assignment of probabilities to alternatives, a social choice correspondence is manipulable. The former definition of manipulability is stronger than the latter, and strategy-proofness based on the former definition is weaker than strategy-proofness based on the latter definition. Duggan and Schwartz (2000) showed that under linear (strict) individual preferences strategy-proof social choice correspondences which satisfy another condition, *residual resoluteness*, is dictatorial, and they mentioned that under the assumption of strict non-imposition, which means that for every alternative there is a profile of individual preferences at which it is chosen by the social choice correspondence as a singleton set, there exists an individual such that at least one of his most preferred alternatives is always chosen by any strategy-proof social choice correspondence.

We will extend their latter result in two ways. First, although they assumed linear orderings of individual preferences (strict preferences), we will consider the case of weak orderings of individual preferences. Second, we will show the existence of an oligarchy for social choice correspondences which satisfy some monotonicity property which we call *modified monotonicity*. An oligarchy is a group of individuals such that it has some decisive power (*semi-decisive*), and at least one of the most preferred alternatives of *every* its member is always chosen by any social choice correspondence. Also we examine the relation between modified monotonicity and strategy-proofness of social choice correspondences according to the definition by Duggan and Schwartz (2000). We will show mainly the following two results. (1) Modified monotonicity implies the existence of an oligarchy. (2) Strategy-proofness of social choice correspondences is equivalent to modified monotonicity.

In the next section we present notations and definitions. In Section 3 we will show the existence of an oligarchy for social choice correspondences which satisfy modified monotonicity. In Section 4 we will show the equivalence of strategy-proofness and modified monotonicity.

2. NOTATIONS, DEFINITIONS AND PRELIMINARY RESULTS

A is the set of alternatives. The number of alternatives is larger than 2. The set of all subsets of A is denoted by $\mathcal{A}$. $N=\{1,2,\ldots,n\}$ is the finite set of individuals with $n\geqslant 2$. Each individual i is endowed with a weak ordering R_i of A. A weak ordering is a reflexive, complete and transitive binary relation. Strict preference and indifference of individual i are denoted by P_i and I_i. Let $\mathcal{R}$ denote the set of all weak orderings of A. A profile p is a function mapping N into the set $\mathcal{R}^n$ of all logically conceivable profiles. For each i p assigns a weak ordering R_i, p' assigns a weak ordering R_i' and so on.

A social choice correspondence is a mapping of $\mathcal{R}^n$ into $\mathcal{A}$. The domain of a social choice correspondence is not restricted (*unrestricted domain*). Given profiles $p, p', p''\ldots$ we denote by $C(p), C(p'), C(p'')\ldots$ the set of alternatives chosen by a social choice correspondence at each profile. We call $C(p)$ the *social choice set* at p and so on.

First we assume strict non-imposition (or citizens' sovereignty) in the following sense.

Strict non-imposition. For every alternative x there is a profile p such that *only* x is chosen by a social choice correspondence, that is, $C(p)=\{x\}$.

This is not a restrictive condition since unanimity implies strict non-imposition. Unanimity is the following condition.

Unanimity. If all individuals most prefer some common alternative, then any social choice correspondence chooses only this alternative.

Now we define modified monotonicity.

Modified monotonicity. Let $C(p)$ be the social choice set at some profile p, and assume that at p for a pair of alternatives $\{x,y\}$ such that $x\in C(p)$ and $y\notin C(p)$:

(1) Individuals in a group $S(S\subseteq N)$: xP_iy for some $x\in C(p)$ or xI_iy for some $x\in C(p)$.
(2) Others (group $S'=N-S$): yP_ix for all $x\in C(p)$.

Consider another profile $p' \in \mathcal{R}^n$ such that individuals in S are partitioned into the following two sub-groups:

(1) S_1: For some common set of alternatives X which includes $C(p)$ and does not include $y(C(p) \subset X$ and $y \notin X)$, $xP'_i z$ for all $x \in X$ and all $z \notin X$,
(2) S_2: Their preferences do no change.

The preferences of individuals in S' at p' are not specified. Then, the social choice correspondence does not choose y at $p'(y \notin C(p'))$.

In another paper, Tanaka (2001), we showed that strategy-proofness according to the definition by Ching and Zhou (2002) is equivalent to a monotonicity property of social choice correspondences which we call *generalized monotonicity*. It is the condition as follows:

Generalized monotonicity. Suppose that there is a profile p such that for a pair of alternatives $\{x, y\}$:

(1) individuals in a group $S(S \subseteq N)$: $xP_i y$
(2) individuals in a group $S'(S' \subseteq N)$: $xI_i y$
(3) individuals in $S''(S' = N - S - S')$: $yP_i x$

and we have $x \in C(p)$ and $y \notin C(p)$. Consider another profile p' such that

(1) individuals in S: $xP'_i y$
(2) individuals in S'': $xP'_i y$ or their preferences are the same as those at p
(3) individuals in S'': not specified

Then, we have $y \neq C(p')$.

In the definition of modified monotonicity we consider the case where at profile p' there is a common set of alternatives, which includes $C(p)$ and does not include y, for individuals in S such that they prefer alternatives in this set to any other alternative. But in the definition of generalized monotonicity we consider only the individual preferences between x and y. Generalized monotonicity implies modified monotonicity. Thus, generalized monotonicity is stronger than modified monotonicity.

An example. Assume that there are three individuals 1, 2, 3. A social choice correspondence (SCC) always chooses one of the most preferred alternatives of individual 1 and one of the most preferred alternatives of individual 2. If their most preferred alternatives are unique and the same, then the SCC chooses only one alternative. This SCC satisfies modified monotonicity but does not satisfy generalized monotonicity.

We can easily verify the following result.

LEMMA 1. *Strict non-imposition and modified monotonicity imply unanimity.*

Further we define the following terminologies.

Semi-decisive. A group of individuals S is *semi-decisive* for x against y if, for any pair of alternatives $\{x, y\}$, when, for some common set of alternatives X such that $x \in X$ and $y \notin X$, individuals in S have preferences xP_iz for all $x' \in X$ and all $z \notin X$, a social choice correspondence does not choose y.

Semi-decisive set. If S is semi-decisive about all pairs of alternatives, it is called a semi-decisive set.

Weak dictator. If a social choice correspondence always chooses at least one of the most preferred alternatives of a particular individual, he is called a weak dictator.

A weak dictator can enforce a social choice correspondence choose his favorite alternative, but he can not exclude his unfavorite alternatives. Therefore he is not a dictator, and there may be multiple weak dictators.

Oligarchy. An oligarchy is a group of individuals such that it is a semi-decisive set and every member of the group is a weak dictator.

3. THE OLIGARCHY THEOREM

In this section we will show that for any social choice correspondence which satisfies strict non-imposition and modified monotonicity there is an oligarchy.

First as a preliminary result we show the following Lemma.

LEMMA 2. *Suppose that a social choice correspondence satisfies strict non-imposition and modified monotonicity.*

(1) *Let partition the individuals into the following two groups, and for alternatives x, y and w we assume the following profile p:*

(i) *individuals in S: $xP_iyP_iwP_iz$*
(ii) *others: $yP_iwP_ixP_iz$*

where z denotes an arbitrary alternative other than x, y and w. Then the social choice correspondence does not choose any alternative other than x and y.

(2) *Similarly, let partition the individuals into the following two groups, and for alternatives x, y and w we assume the following profile p:*

(i) *individuals in S: $wP_ixP_iyP_iz$*
(ii) *others: $yP_iwP_ixP_iz$*

where z denotes an arbitrary alternative other than x, y and w. Then the social choice correspondence does not choose any alternative other than y and w.

Proof. See Appendix A. □

Next, using this lemma, we show the following result. This is key to our conclusion.

LEMMA 3. *Suppose that a social choice correspondence satisfies strict non-imposition and modified monotonicity. If there exists a group of individuals which is semi-decisive about one pair of alternatives, then it is semi-decisive about all pairs of alternatives, that is, it is a semi-decisive set.*

Proof. Suppose that a group S is semi-decisive for x against y. Let w be an alternative other than x and y, and denote each other alternative by z.

(1) Consider the following profile p:

(i) individuals in S: $xP_iyP_iwP_iz$
(ii) others: $yP_iwP_ixP_iz$

Since S is semi-decisive for x against y, we have $y \notin C(p)$. From (1) of Lemma 2 we have $w \notin C(p)$ and $z \notin C(p)$, and so we have $C(p) = \{x\}$. Individuals in S prefer x to w, but all other individuals prefer w to x. Therefore, by modified monotonicity S is semi-decisive for x against w.

(2) Next consider the following profile p':

(i) individuals in S: $wP'_i xP'_i yP'_i z$
(ii) others: $yP'_i wP'_i xP'_i z$

Since S is semi-decisive for x against y, we have $y \notin C(p')$. From (2) of Lemma 2 we have $x \notin C(p')$ and $z \notin C(p')$, and so we have $C(p') = \{w\}$. Individuals in S prefer w to y, but all other individuals prefer y to w. Therefore, by modified monotonicity S is semi-decisive for w against y.

Applying this logic repeatedly, the proof of this lemma will be completed. □

Next we show

LEMMA 4. *Suppose that there are two semi-decisive sets. Then, the set which is the intersection of these two sets is also a semi-decisive set.*

Proof. The proof of this lemma is similar to that of Theorem 3.6 in Craven (1992).

See Appendix B. □

From this lemma, Lemma 3 and modified monotonicity the set of all individuals is a semi-decisive set. Since the number of individuals is finite, there exists a minimum semi-decisive set. *Minimum* means that the number of individuals included in the set is minimum among all semi-decisive sets. By the definition of semi-decisive sets and Lemma 4 we can easily verify the following results.

LEMMA 5. (1) *We can not have multiple disjoint semi-decisive sets.*
(2) *There does not exist multiple different minimum semi-decisive sets.*

Therefore the minimum semi-decisive set is unique.

Now we can show the following theorem.

THEOREM 1. *Every individual included in the minimum semi-decisive set is a weak dictator.*

Proof. If the minimum semi-decisive set includes only one individual, then he is the dictator, and the dictator is a weak dictator. Assume that the minimum semi-decisive set includes at least two individuals, denote it by S, and consider the following profile:

(1) An individual included in S (denoted by j): $y'P_jxP_jwP_jz$ for all $y' \in Y$ for a set of alternatives Y, and he is indifferent between any pair of alternatives included in Y
(2) Individuals in S other than j: $xP_iwP_izP_iy'$ for all $y' \in Y$
(3) Others: $wP_ixP_izP_iy'$ for all $y' \in Y$

where z denotes an arbitrary alternative other than x, w and alternatives in Y.

(1) means that Y is the set of the most preferred alternatives for individual j. Since S is a semi-decisive set, the social choice correspondence does not choose w and z, and choose x or some alternatives in Y. If only one alternative in Y is chosen, since only individual j prefers this alternative to x, and all other individuals prefer x to it, individual j is the dictator by modified monotonicity and Lemma 3. If only x is chosen, since only individuals other than j prefer x to alternatives in Y, and individual j prefers alternatives in Y to x, the group of individuals other than j is a semi-decisive set by modified monotonicity and Lemma 3. Then, from Lemma 4 the intersection of this group and S is also a semi-decisive set. It is a group of individuals included in S other than $j(S-\{j\})$. It contradicts the assumption that S is the minimum semi-decisive set. Therefore, x and at least one alternative in Y, or at least two alternatives in Y are chosen by the social choice correspondence.

Assume that the preference of individual j changes maintaining the property that he prefers alternatives in Y to x, w and z. Then, from modified monotonicity it is impossible that none of alternatives in Y is chosen by the social choice correspondence. Given a preference of individual j, if, when the preference of one of the other individuals changes, none of alternatives in Y is not chosen, then modified monotonicity implies that any alternative in Y is not chosen before the change of his preference because he prefers x, w and z to all alternatives in Y before the change of his preference. It is a contradiction. Therefore, so long as Y is the set of the most preferred alternatives of individual j, the social choice correspondence chooses at least one alternative in Y.

Since j and Y are arbitrary, every individual in S is a weak dictator. □

This theorem implies that the minimum semi-decisive set is an oligarchy.

4. EQUIVALENCE OF MODIFIED MONOTONICITY AND STRATEGY-PROOFNESS

In this section we will show the equivalence of modified monotonicity and strategy-proofness according to the definition by Duggan and Schwartz (2000). We assume that each individual (represented by i) has a von Neumann-Morgenstern utility function u_i. If we have $u_i(x) > u_i(y)$ when xP_iy and $u_i(x) = u_i(y)$ when xI_iy, the preference of individual i *is represented by* u_i.

Let p and p' be two profiles between which only the preference of individual i is different. $C(p)$ and $C(p')$ are the social choice sets at p and p'. Assume that individual i assigns the probability $\mu(x)$ and $\mu'(x)$ to an alternative x included in $C(p)$ and $C(p')$, and so on. $\mu(x)$ is individual i's subjective probability of alternative x when $C(p)$ is the social choice set, and similarly $\mu'(x)$ is his subjective probability when $C(p')$ is the social choice set. Then, his expected utilities at p and p' evaluated by his utility function at p are

$$E_i(p) = \frac{1}{\sum_{x\in C(p)} \mu(x)} \sum_{x\in C(p)} \mu(x) u_i(x)$$

and

$$E_i(p') = \frac{1}{\sum_{x\in C(p')} \mu'(x)} \sum_{x\in C(p')} \mu'(x) u_i(x)$$

If for *all* assignments of probabilities to alternatives we have

$$E_i(p') > E_i(p), \tag{1}$$

then individual i has an incentive to report his preference R_i' when his true preference is R_i, and the social choice correspondence is manipulable by individual i at p. Conversely, if for *some* assignment of probabilities we have $E_i(p) \geqslant E_i(p')$, the social choice correspondence is not manipulable.

Now we can show the following Lemma.

LEMMA 6. *Let p and p' be two profiles of individual preferences between which only the preference of individual i is different. If and only if for some $x \in C(p')$ and all $y \in C(p)$, or for some $y \in C(p)$ and all $x \in C(p')$ individual i has a preference xP_iy, the social choice correspondence is manipulable by individual i at p.*

Proof. First consider the case where for some $x \in C(p')$ and all $y \in C(p)$ individual i has a preference xP_iy. Let $\varepsilon > 0$ be the probability of x assigned by individual i at p', w be one of the top-ranked (most preferred) alternatives for him in $C(p)$, v be one of the bottom-ranked (least preferred) alternatives for him in $C(p')$ evaluated by his utility function at p, u_i. Then we obtain

$$E_i(p') \geqslant \varepsilon u_i(x) + (1-\varepsilon)u_i(v)$$

and

$$E_i(p) \leqslant u_i(w)$$

Since $u_i(x) > u_i(w)$ and $u_i(x) \geqslant u_i(v)$, given ε we can select the value of $u_i(x)$ such that $E_i(p') > E_i(p)$ holds.

Next consider the case where for some $y \in C(p)$ and all $x \in C(p')$ individual i has a preference xP_iy. Let $\varepsilon > 0$ be the probability of y assigned by individual i at p, w be one of the bottom-ranked (least preferred) alternatives for him in $C(p')$, and v be one of the top-ranked (most preferred) alternatives for him in $C(p)$ evaluated by his utility function at p, u_i. Then we obtain

$$E_i(p') \geqslant u_i(w)$$

and

$$E_i(p) \leqslant \varepsilon u_i(y) + (1-\varepsilon)u_i(v)$$

Since $u_i(y) < u_i(w)$ and $u_i(y) \leqslant u_i(v)$, given ε we can select the value of $u_i(y)$ such that $E_i(p') > E_i(p)$ holds.

Finally, assume that there exits no $x \in C(p')$ such that xP_iy for all $y \in C(p)$, and no $y \in C(p)$ such that xP_iy for all $x \in C(p')$. Let x be one of the top-ranked alternatives for individual i in $C(p')$ and y be one of his bottom-ranked alternatives in $C(p)$ evaluated by his utility function at p, u_i. Then, there exists at least one $w \in C(p)$ such that wR_ix and at least one $z \in C(p')$ such that yR_iz. Let ε' and ε be, respectively, the probability of z at p' and the probability of w at p assigned by individual i. Then we obtain

$$E_i(p') \leqslant \varepsilon' u_i(z) + (1 - \varepsilon')u_i(x)$$

and

$$E_i(p) \geqslant \varepsilon u_i(w) + (1 - \varepsilon)u_i(y)$$

Since $u_i(w) \geqslant u_i(x)$ and $u_i(y) \geqslant u_i(z)$, if we assume $\varepsilon = 1 - \varepsilon'$, we obtain $E_i(p) \geqslant E_i(p')$, and (1) does not hold. □

Strategy-proofness is defined as follows:

Strategy-proofness. If a social choice correspondence is not manipulable for any individual at any profile, it is *strategy-proof.*

Now we show the following theorem.

THEOREM 2. *Modified monotonicity and strategy-proofness according to Duggan and Schwartz* (2000) *are equivalent.*

In the following proof we use notations in the definition of modified monotonicity, and we neglect individuals in S_2 whose preferences do not change between p and p'.

Proof. (1) First we show that strategy-proofness of social choice correspondences implies modified monotonicity. Let individuals 1 to m $(0 \leqslant m \leqslant n)$ belong to S and individuals $m+1$ to n belong to S'. Consider a profile p'' other than p and p' such that individuals in S have a preference $xP''_iyP''_iz$ for all $x \in C(p)$, and individuals in S' have a preference $yP''_ixP''_iz$ for all $x \in C(p)$, where z is an arbitrary alternative other than alternatives in $C(p)$ and y.

Let p^1 be the profile such that only the preference of individual 1 changes from R_1 to R_1'', and suppose that at p^1 an alternative other than alternatives in $C(p)$ is included in the social choice set. Then, he has an incentive to report a false preference R_1 when his true preference is R_1'' because he prefers alternatives in $C(p)$ to all other alternatives at p^1. Therefore, at p^1 only alternatives in $C(p)$ are chosen by the social choice correspondence. By the same logic, when the preferences of individuals 1 to m change from R_i to R_i'', only alternatives in $C(p)$ are chosen. Next, let p^{m+1} be the profile such that the preference of individual $m+1$, as well as the preferences of the first m individuals, changes from R_{m+1} to R_{m+1}'', and suppose that at $p^{m+1} y$ is included in the social choice set. Then, he has an incentive to report a false preference R_{m+1}'' when his true preference is R_{m+1} because his preference at p is $yP_{m+1}x$ for all $x \in C(p)$. On the other hand, if an alternative other than alternatives in $C(p)$ is included in the social choice set at p^{m+1}, he has an incentive to report a false preference R_{m+1} when his true preference is R_{m+1}'' because his preference at p^{m+1} is $xP_{m+1}''z$ for all $x \in C(p)$ and all $z \notin C(p)$, $z \neq y$. By the same logic, when the preferences of all individuals change from R_i to R_i'', only alternatives in $C(p)$ are chosen by the social choice correspondence.

Now, suppose that from p'' to p' the individual preferences change one by one from R_i'' to R_i'. If, when the preference of the first individual in S_1 changes, an alternative outside X is chosen, he has an incentive to report a false preference R_i'' when his true preference is R_i' because his preference at p' is $xP_i'z$ for all $x \in X$ and all $z \notin X$. Consequently only some alternatives included in X are chosen. By the same logic, when the preferences of all individuals in S_1 change from R_i to R_i'', only some alternatives in X are chosen. Further, if, when the preference of the first individual (individual $m+1$) in S' changes, y is included in the social choice set, then he has an incentive to report a false preference R_{m+1}' when his true preference is R_{m+1}'' because his preference at p'' is $yP_{m+1}''z$ for all $z \neq y$. By the same logic, when the preferences of all individuals change, y is not chosen by the social choice correspondence.

(2) Denote the social choice sets at profiles p and p' by $C(p)$ and $C(p')$. Between p and p' only the preference of individual i is different. Assume that a social choice correspondence which

satisfies modified monotonicity is manipulable. Then, there is a case where, either of the following (i) or (ii) holds.

(i) For some $x \in C(p)$ and all $y \in C(p')$ individual i's preference is yP_ix.
(ii) For some $y \in C(p')$ and all $x \in C(p)$ individual i's preference is yP_ix.

First consider (i). From the assumption we have $x \notin C(p')$. Comparing p' and p, since individual i has a preference yP_ix for all $y \in C(p')$ at p, individuals, whose preferences are yR'_ix for some y at p', prefer y to x at p (when individual i's preference is yR'_ix), or their preferences do not change. Consequently the assumptions for modified monotonicity are satisfied, and x is not chosen by the social choice correspondence at p. It contradicts the assumption.

Next consider (ii). From the assumption we have $y \notin C(p)$. Comparing p and p', since individual i has a preference yP_ix at p, the preferences of individuals, whose preferences at p are xR_iy for some x, do not change. Consequently the assumptions for modified monotonicity are satisfied, and y is not chosen by the social choice correspondence at p'. It contradicts the assumption. □

5. CONCLUDING REMARKS

We have presented an analysis of strategy-proof social choice correspondences and proved an impossibility theorem (the oligarchy theorem). There are several definitions of strategy-proofness of social choice correspondences. We hope to get a unified view of strategy-proofness of such social choice rules and several impossibility theorems in the future research.

APPENDICES

A. Proof of Lemma 2

(1) By strict non-imposition there is a profile p' at which $C(p') = \{y\}$. Suppose that, staring from individuals other than those in S, their preferences change from R'_i to R_i (from profile p' to p)

one by one. Even when the preferences of individuals outside S change, only y is chosen because they strictly most prefer y at p. On the other hand, when the preferences of individuals in S change, any alternative other than x and y is not chosen because they most prefer x and secondly prefer y at p.

(2) Permuting w, x and y and exchanging S and N/S, this case is identical to the first case.

B. Proof of Lemma 4

Denote two semi-decisive sets by S_1 and S_2, and the group which is the intersection of S_1 and S_2 by S_3. Consider the following profile p:

(1) individuals in $S_1 \backslash S_3$: $xP_i yP_i wP_i z$
(2) individuals in S_3: $wP_i xP_i yP_i z$
(3) individuals in $S_2 \backslash S_3$: $yP_i wP_i xP_i z$
(4) others: $yP_i xP_i wP_i z$

Since S_1 is a semi-decisive set, we have $y \notin C(p)$ and $z \notin C(p)$. Similarly, since S_2 is a semi-decisive set, we have $x \notin C(p)$. Then, we have $C(p) = \{w\}$. Since only individuals in S_3 prefer w to y, and all other individuals prefer y to w, S_3 is a semi-decisive set (it is semi-decisive for w against y) by modified monotonicity and Lemma 3.

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NOTE

1. There are several classical works about strategy-proofness of social choice correspondences such as Barbera (1977), Kelly (1977), Feldman (1980), Gärdenfors (1976) and (1979).

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