Profitability of price and quantity strategies in an oligopoly

Yasuhito Tanaka*

Faculty of Law, Chuo University, 742-1 Higashinakano, Hachioji, Tokyo 192-0393, Japan

Abstract

In this paper, we examine a sub-game perfect equilibrium of a two stage game in an oligopoly in which n ($n \geq 2$) firms produce differentiated (substitutable) goods. In the first stage of the two stage game, the firms choose their strategic variable, price or quantity. In the second stage, they choose the levels of their strategic variables. We will show the following results. A quantity strategy is the best response for each firm when all other firms choose a price strategy. Thus, the Bertrand equilibrium does not constitute a sub-game perfect equilibrium of the two stage game. A quantity strategy is also the best response for each firm when all other firms choose a quantity strategy. Therefore, the Cournot equilibrium constitutes a sub-game perfect equilibrium. © 2001 Elsevier Science B.V. All rights reserved.

1. Introduction

In this paper, we examine a sub-game perfect equilibrium of a two stage game in an oligopoly in which n ($n \geq 2$) firms produce differentiated (substitutable) goods. In the first stage of the two stage game, the firms choose their strategic variable, price or quantity. In the second stage, they choose the levels of their strategic variables.

Singh and Vives (1984) showed that in a duopoly with substitutable goods in which firms can choose a price or quantity strategy, a quantity strategy dominates a price strategy, and

* Fax: +81-426-74-3202.

E-mail address: yasuhito@tamacc.chuo-u.ac.jp (Y. Tanaka).

the Cournot equilibrium constitutes a sub-game perfect equilibrium of the two stage game. [1] In this paper, we will extend their analysis to an oligopoly.

In the following sections, we will show the following results. A quantity strategy is the best response for each firm when all other firms choose a price strategy. Thus, the Bertrand equilibrium does not constitute a sub-game perfect equilibrium of the two stage game. A quantity strategy is also the best response for each firm when all other firms choose a quantity strategy. Therefore, the Cournot equilibrium constitutes a sub-game perfect equilibrium.

2. The model and analyses

We consider a sub-game perfect equilibrium of a two stage game of an oligopoly in which n ($n \geq 2$) firms produce substitutable goods. Two stages of the game are as follows.

1. In the first stage, the firms choose their strategic variable, price or quantity.
2. In the second stage, they choose the levels of their strategic variables.

The firms are identical except for their strategy choice, and (direct and inverse) demand functions are symmetric.

We consider four equilibrium configurations in the second stage of the game corresponding to the strategy choice of the firms in the first stage as follows.

1. *The Bertrand equilibrium*: All firms choose a price strategy in the first stage.
2. *The quasi Bertrand equilibrium*: Only one of the firms chooses a quantity strategy, and other $n - 1$ firms choose a price strategy in the first stage.
3. *The Cournot equilibrium*: All firms choose a quantity strategy in the first stage.
4. *The quasi Cournot equilibrium*: Only one of the firms chooses a price strategy, and other $n - 1$ firms choose a quantity strategy in the first stage.

We will show the following results. (1) The profit of the firm who chooses a quantity strategy in the quasi Bertrand equilibrium is *larger* than its profit in the Bertrand equilibrium. Thus, a quantity strategy is the best response for each firm when all other firms choose a price strategy. (2) The profit of the firm who chooses a price strategy in the quasi Cournot equilibrium is *smaller* than its profit in the Cournot equilibrium. Thus, a quantity strategy is also the best response for each firm when all other firms choose a quantity strategy.

2.1. Bertrand and quasi Bertrand equilibria

The set of all firms is denoted by *N*, and one of the firms who may choose a price strategy or a quantity strategy is called, without loss of generality, *Firm k*. The rest of the firms are called *the other firms*. They choose a price strategy.

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[1] Cheng (1985) presented a geometric analysis, and Jéhiel and Walliser (1995) generalized Singh and Vives' analysis to a general two person game. Other related papers are Vives (1985), Klemperer and Meyer (1986), Okuguchi (1987) and Qin and Stuart (1997). Klemperer and Meyer (1986) analyzed the Nash equilibria of a one stage game, not two stage game, in which strategic variables are determined endogenously. Qin and Stuart (1997) considered the choice of strategic variable in a homogeneous oligopoly.

The (direct) demand functions are

$$q_i = h_i(\boldsymbol{p}), \quad \text{for } i \in N, \tag{1}$$

where p_i and q_i denote the price and output of each firm, and $\boldsymbol{p} = \{p_1, p_2, \ldots, p_n\}$.

Similarly to the assumptions in Vives (1985), we assume the following assumption.

Assumption 1.

1. $h_i(\boldsymbol{p})$ is a continuous function on R_+^n, and symmetric for all firms.
2. $h_i(\boldsymbol{p})$ is positive in a non-empty bounded region of R_+^n, X_i. Let $X = \bigcap_{i=1}^n X_i$.
3. $h_i(\boldsymbol{p})$ is continuously twice differentiable in int X, and is decreasing in p_i and increasing in p_j, $j \neq i$, that is, $(\partial h_i / \partial p_i) < 0$ and $(\partial h_i / \partial p_j) > 0$, for all $i, j \in N$, $j \neq i$, because the goods are substitutes.

All firms have the same cost function $c(q_i)$ with $c'(q_i) > 0$. The profits of the firms are

$$\pi_i = p_i h_i(\boldsymbol{p}) - c(h_i(\boldsymbol{p})), \quad \text{for } i \in N. \tag{2}$$

First, we consider the Bertrand equilibrium in which all firms choose a price strategy. They determine the prices of their goods given the prices of the other firms' goods. Thus, they are price takers. The first-order conditions for profit maximization are

$$\frac{\partial \pi_i}{\partial p_i} = h_i(\boldsymbol{p}) + [p_i - c'(h_i(\boldsymbol{p}))] \frac{\partial h_i(\boldsymbol{p})}{\partial p_i} = 0, \quad \text{for } i \in N. \tag{3}$$

Now, we assume the following assumption.

Assumption 2.

$$p_i - c'(h_i(\boldsymbol{p})) > 0, \quad \text{and} \quad \frac{\partial^2 \pi_i}{\partial p_i^2} + \left| \sum_{j=1, j \neq i}^n \frac{\partial^2 \pi_i}{\partial p_i p_j} \right| < 0,$$

for $i \in N$, $j \neq i$, for all $\boldsymbol{p} \in X$.

This assumption means that the effect of a change in the price of one firm on the marginal profit (with respect to price) of this firm is larger than the sum of the effects of changes in the prices of other firms. It ensures that there exist a unique Bertrand equilibrium (Friedman, 1977, 1983).

The second-order condition for each firm, $(\partial^2 \pi_i / \partial p_i^2) < 0$ and Assumption 2 with the symmetry of the oligopoly imply

$$\frac{\partial^2 \pi_i}{\partial p_i^2} + (n-2) \frac{\partial^2 \pi_i}{\partial p_i p_j} < 0, \quad \text{for } i \in N, \; j \neq i. \tag{4}$$

We consider a reaction function of Firm k and a reaction function of the other firms given all p_i's, $i \neq k$, are equal, that is, a reaction function of Firm k to a simultaneous and equal change in the prices of the other firms and a reaction function of the common price of the

other firms to a change in p_k. We call these reaction functions *the Bertrand reaction function* of Firm k and that of the other firms, and denote them by $p_k = R_k^B(p_i)$ and $p_i = R_i^B(p_k)$. The slope of the Bertrand reaction function of Firm k is equal to

$$R_k'^B(p_i) = \left. \frac{\mathrm{d}p_k}{\mathrm{d}p_i} \right|_{\text{all } p_i,\, i \neq k,\, \text{are equal}} = (n-1)\frac{\mathrm{d}p_k}{\mathrm{d}p_i} = -\frac{(n-1)(\partial^2\pi_k/\partial p_k\, p_i)}{\partial^2\pi_k/\partial p_k^2},$$

and the slope of the Bertrand reaction function of the other firms is

$$R_i'^B(p_k) = \left. \frac{\mathrm{d}p_i}{\mathrm{d}p_k} \right|_{\text{all } p_i,\, i \neq k,\, \text{are equal}} = -\frac{\partial^2\pi_i/\partial p_k\, p_i}{(\partial^2\pi_i/\partial p_i^2) + (n-2)(\partial^2\pi_i/\partial p_i\, p_j)},$$
$$\text{for } i \in N,\ i \neq k,\ j \neq i, k. \qquad (5)$$

From Eq. (4), the denominator of Eq. (5) is negative.

Assumption 2 does not restrict the sign of the reaction functions. Since the goods are substitutes, we assume the following assumption.

Assumption 3. The Bertrand reaction functions are upward sloping.

This assumption is equivalent to [2]

$$\frac{\partial^2\pi_i}{\partial p_i\, p_j} > 0, \quad \text{for } i \in N,\ j \neq i. \qquad (6)$$

From Assumption 2 and Eq. (6), we have

$$\frac{\partial^2\pi_i}{\partial p_i^2} + \left| (n-1)\frac{\partial^2\pi_i}{\partial p_i\, p_j} \right| < 0,$$

and

$$\frac{\partial^2\pi_i}{\partial p_i^2} + \left| (n-2)\frac{\partial^2\pi_i}{\partial p_i\, p_j} \right| < - \left| \frac{\partial^2\pi_i}{\partial p_i\, p_j} \right| < 0.$$

Since $(\partial^2\pi_i/\partial p_i^2) < 0$, these inequalities imply

$$\left| \frac{\partial^2\pi_i}{\partial p_i^2} \right| > \left| (n-1)\frac{\partial^2\pi_i}{\partial p_i\, p_j} \right|,$$

and

$$\left| \frac{\partial^2\pi_i}{\partial p_i^2} + (n-2)\frac{\partial^2\pi_i}{\partial p_i\, p_j} \right| > \left| \frac{\partial^2\pi_i}{\partial p_i\, p_j} \right|.$$

Thus, the magnitudes of the slopes of the Bertrand reaction functions are smaller than one.

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[2] With Eq. (6), an individual reaction function of each firm, whose slope is $(\mathrm{d}p_i/\mathrm{d}p_j) = -(\partial^2\pi_i/\partial p_i\, p_j)/(\partial^2\pi_i/\partial p_i^2)$, $j \neq i$, is also upward sloping.

Next, suppose that Firm k changes its strategy from price to quantity. Firm k determines the output of its good given the prices of the other firms' goods, and it is still a price taker. Given the prices of the other firms whether Firm k determines its price or output is irrelevant. Thus, its profit maximization condition and reaction function are the same as in the pure Bertrand case considered above. On the other hand, each of the other $n-1$ firms determines the price of its good given the prices of the other $n-2$ firms' goods, and the output of Firm k's good. Thus, they are partially price takers and partially quantity takers. We call the equilibrium in this case *the quasi Bertrand equilibrium*.

The first-order condition for Firm k is the same as Eq. (3). The first-order conditions for the other firms are

$$\frac{\partial \pi_i}{\partial p_i} + \frac{\partial \pi_i}{\partial p_k}\frac{\mathrm{d}p_k}{\mathrm{d}p_i} = h_i(\boldsymbol{p}) + [p_i - c'(h_i(\boldsymbol{p}))]\frac{\partial(h_i(\boldsymbol{p}))}{\partial p_i}$$

$$-[p_i - c'(h_i(\boldsymbol{p}))]\frac{\partial(h_i(\boldsymbol{p}))}{\partial p_k}\frac{\partial^2 \pi_k/\partial p_k p_i}{\partial^2 \pi_k/\partial p_k^2} = 0,$$

$$\text{for } i \in N, \ i \neq k. \tag{7}$$

We consider a reaction function of the other firms given all p_i, $i \neq k$, are equal, that is, a reaction function of the common price of the other firms to a change in p_k in the price space ($p_i - p_k$ space). We call it *the quasi Bertrand reaction function* of the other firms, and denote it by $p_i = \bar{R}_i^B(p_k)$

Now, we will show the following lemma.

Lemma 1. *The quasi Bertrand reaction function of the other firms lies more outward than their Bertrand reaction function in the price space, that is, $\bar{R}_i^B(p_k) > R_i^B(p_k)$ given p_k.*

Proof. When Eq. (7) holds, the left-hand side of Eq. (3) is equal to

$$[p_i - c'(h_i(\boldsymbol{p}))]\frac{\partial(h_i(\boldsymbol{p}))}{\partial p_k}\frac{\partial^2 \pi_k/\partial p_k p_i}{\partial^2 \pi_k/\partial p_k^2} < 0.$$

From Eq. (4), we know that a simultaneous and equal increase in the prices of the other firms reduces the marginal profit of each other firm, $\partial \pi_i/\partial p_i$, $i \neq k$. Thus, p_i in the Bertrand reaction function of the other firms is smaller than p_i in their quasi Bertrand reaction function given p_k and $\bar{R}_i^B(p_k) > R_i^B(p_k)$. $\qquad\square$

Then, we obtain the following result.

Theorem 1. *The profit of Firm k in the quasi Bertrand equilibrium is larger than its profit in the Bertrand equilibrium.*

Proof. See Appendix A. $\qquad\square$

From this theorem, we find the following corollary.

Corollary 1. *A quantity strategy is the best response for each firm when all other firms choose a price strategy. Thus, the Bertrand equilibrium does not constitute a sub-game perfect equilibrium of the two stage game.*

2.2. *Cournot and quasi Cournot equilibria*

We use similar notation as in the previous subsection. The set of all firms is denoted by N, and one of the firms who may choose a price strategy or a quantity strategy is called Firm k. The rest of the firms are called the other firms. They choose a quantity strategy.

The inverse demand functions, which are derived from the demand functions in Eq. (1) are

$$p_i = f_i(\boldsymbol{q}), \quad \text{for } i \in N,$$

where $\boldsymbol{q} = \{q_1, q_2, \ldots, q_n\}$.

Similarly to Assumption 1, we assume the following assumption.

Assumption 4.

1. $f_i(\boldsymbol{q})$ is a continuous function on R_+^n, and symmetric for all firms.
2. $f_i(\boldsymbol{q})$ is positive in a non-empty bounded region of R_+^n, Y_i. Let $Y = \bigcap_{i=1}^{n} Y_i$.
3. $f_i(\boldsymbol{q})$ is continuously twice differentiable in int Y, and is decreasing in its all arguments, that is, $(\partial f_i / \partial q_i) < 0$ and $(\partial f_i / \partial q_j) < 0$, for all $i, j \in N$, $j \neq i$, because the goods are substitutes.

The profits of the firms are

$$\pi_i = f_i(\boldsymbol{q})q_i - c(q_i), \quad \text{for } i \in N. \tag{8}$$

First, consider the Cournot equilibrium in which all firms choose a quantity strategy. They determine the outputs of their goods given the outputs of the other firms' goods. Thus, they are quantity takers. The first-order conditions for profit maximization are

$$\frac{\partial^2 \pi_i}{\partial q_i} = f_i(\boldsymbol{q}) + q_i \frac{\partial f_i(\boldsymbol{q})}{\partial q_i} - c'(q_i) = 0, \quad \text{for } i \in N. \tag{9}$$

Now, we assume the following assumption.

Assumption 5.

$$\frac{\partial^2 \pi_i}{\partial q_i^2} + \left| \sum_{j=1, j \neq i}^{n} \frac{\partial^2 \pi_i}{\partial q_i q_j} \right| < 0, \quad \text{for } i \in N, \ j \neq i, \text{ for all } \boldsymbol{q} \in Y.$$

This assumption means that the effect of a change in the output of one firm on the marginal profit (with respect to output) of this firm is larger than the effects of changes in the outputs of other firms. It ensures that there exist a unique Cournot equilibrium (Friedman, 1977, 1983).

The second-order condition for each firm, $(\partial^2 \pi_i / \partial q_i^2) < 0$ and Assumption 5 with the symmetry of the oligopoly imply

$$\frac{\partial^2 \pi_i}{\partial q_i^2} + (n - 2)\frac{\partial^2 \pi_i}{\partial q_i q_j} < 0, \quad \text{for } i \in N, \ j \neq i. \tag{10}$$

We consider a reaction function of Firm k and a reaction function of the other firms given all q_i, $i \neq k$, are equal, that is, a reaction function of Firm k to a simultaneous and equal change in the outputs of the other firms and a reaction function of the common output of the other firms to a change in q_k. We call these reaction functions *the Cournot reaction function* of Firm k and that of the other firms, and denote them by $q_k = R_k^C(q_i)$ and $q_i = R_i^C(q_k)$. The slope of the Cournot reaction function of Firm k is equal to

$$
R_k'^C(q_i) = \left. \frac{\mathrm{d}q_k}{\mathrm{d}q_i} \right|_{\text{all } q_i, \, i \neq k, \text{ are equal}} = (n-1)\frac{\mathrm{d}q_k}{\mathrm{d}q_i} = -\frac{(n-1)(\partial^2 \pi_k / \partial q_k q_i)}{\partial^2 \pi_k / \partial q_k^2},
$$

and the slope of the Cournot reaction function of the other firms is

$$
R_i'^C(q_k) = \left. \frac{\mathrm{d}q_i}{\mathrm{d}q_k} \right|_{\text{all } q_i, \, i \neq k, \text{ are equal}} = -\frac{\partial^2 \pi_i / \partial q_i \, p_k}{(\partial^2 \pi_i / \partial q_i^2) + (n-2)(\partial^2 \pi_i / \partial q_i q_j)},
$$
$$
\text{for } i \in N, \; i \neq k, \; j \neq i, k. \tag{11}
$$

From Eq. (10), the denominator of Eq. (11) is negative.

Assumption 5 does not restrict the sign of the reaction functions. Since the goods are substitutes, we assume the following assumption.

Assumption 6. The Cournot reaction functions are downward sloping.

This assumption is equivalent to [3]

$$
\frac{\partial^2 \pi_i}{\partial q_i q_j} < 0, \quad \text{for } i \in N, \; j \neq i. \tag{12}
$$

From Assumption 5 and Eq. (12), we have

$$
\frac{\partial^2 \pi_i}{\partial q_i^2} + \left| (n-1)\frac{\partial^2 \pi_i}{\partial q_i q_j} \right| < 0,
$$

and

$$
\frac{\partial^2 \pi_i}{\partial q_i^2} + \left| (n-2)\frac{\partial^2 \pi_i}{\partial q_i q_j} \right| < - \left| \frac{\partial^2 \pi_i}{\partial q_i q_j} \right| < 0.
$$

Since $(\partial^2 \pi_i / \partial q_i^2) < 0$, these inequalities imply

$$
\left| \frac{\partial^2 \pi_i}{\partial q_i^2} \right| > \left| (n-1)\frac{\partial^2 \pi_i}{\partial q_i q_j} \right|,
$$

and

$$
\left| \frac{\partial^2 \pi_i}{\partial q_i^2} + (n-2)\frac{\partial^2 \pi_i}{\partial q_i q_j} \right| > \left| \frac{\partial^2 \pi_i}{\partial q_i q_j} \right|.
$$

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[3] With Eq. (12), an individual reaction function of each firm, whose slope is $(\mathrm{d}q_i / \mathrm{d}q_j) = -(\partial^2 \pi_i / \partial q_i q_j)/(\partial^2 \pi_i / \partial q_i^2)$, $j \neq i$, is also downward sloping.

Thus, the magnitudes of the slopes of the Cournot reaction functions are smaller than one.

Next, suppose that Firm k changes its strategy from quantity to price. Firm k determines the price of its good given the outputs of the other firms' goods, and it is still a quantity taker. Given the outputs of the other firms whether Firm k determines its price or output is irrelevant. Thus, its profit maximization condition and reaction function are the same as in the pure Cournot case considered above. On the other hand, each of the other $n - 1$ firms determines the output of its good given the outputs of the other $n - 2$ firms' goods, and the price of Firm k's good. Thus, they are partially quantity takers and partially price takers. We call the equilibrium in this case *the quasi Cournot equilibrium*.

The first-order condition for Firm k is the same as Eq. (9). The first-order conditions for the other firms are

$$
\frac{\partial \pi_i}{\partial q_i} + \frac{\partial \pi_j}{\partial q_i} \frac{\mathrm{d}q_j}{\mathrm{d}q_i} = f_i(\boldsymbol{q}) + q_i \frac{\partial f_i(\boldsymbol{q})}{\partial q_i} - q_i \frac{\partial f_i(\boldsymbol{q}))}{\partial q_j} \frac{\partial^2 \pi_j / \partial q_j q_i}{\partial^2 \pi_j / \partial q_k^2} - c'(q_i) = 0,
$$
$$
\text{for } i \in N, \ i \neq k. \tag{13}
$$

We consider a reaction function of the other firms given all q_i, $i \neq k$, are equal, that is, a reaction function of the common output of the other firms to a change in q_k in the quantity space ($q_i - q_k$ space). We call it *the quasi Cournot reaction function* of the other firms, and denote it by $q_i = \bar{R}_i^C(q_k)$.

Now, we will show the following lemma.

Lemma 2. *The quasi Cournot reaction function of the other firms lies more outward than their Cournot reaction function in the quantity space, that is, $\bar{R}_i^C(q_k) > R_i^C(q_k)$ given q_k.*

Proof. When Eq. (13) holds, the left-hand side of Eq. (9) is equal to

$$
q_i \frac{\partial f_i(\boldsymbol{q})}{\partial q_k} \frac{\partial^2 \pi_k / \partial q_k q_i}{\partial^2 \pi_k / \partial q_k^2} < 0.
$$

From Eq. (10), we know that a simultaneous and equal increase in the outputs of the other firms reduces the marginal profit of each other firm, $\partial \pi_i / \partial q_i$, $i \neq k$. Thus, q_i in the Cournot reaction function of the other firms is smaller than q_i in their quasi Cournot reaction function given q_k, and $\bar{R}_i^C(q_k) > R_i^C(q_k)$. □

Then, we obtain the following result.

Theorem 2. *The profit of Firm k in the Cournot equilibrium is larger than its profit in the quasi Cournot equilibrium.*

Proof. See Appendix B. □

From this theorem, we find the following corollary.

Corollary 2. *A quantity strategy is the best response for each firm when all other firms choose a quantity strategy. Therefore, the Cournot equilibrium constitutes a sub-game perfect equilibrium of the two stage game.*

3. Conclusion

In this paper, we have examined the choice of strategy by the firms, price or quantity, in an oligopoly with substitutable goods. We have shown that a quantity strategy is more profitable than a price strategy for each firm when all other firms choose a price strategy or when all other firms choose a quantity strategy. Thus, the Cournot equilibrium constitutes a sub-game perfect equilibrium, and the Bertrand equilibrium does not.

Acknowledgements

Appendix A

Proof of Theorem 1. The quasi Bertrand reaction function of the other firms lies more outward than their Bertrand reaction function. The Bertrand reaction function of Firm k and that of the other firms are upward sloping, and the magnitude of the slope of these Bertrand reaction functions is smaller than one. In the quasi Bertrand equilibrium, the price of Firm k's good is determined along its Bertrand reaction function, on the other hand the common price of the goods of the other firms is determined along their *quasi* Bertrand reaction function. Thus, in the quasi Bertrand equilibrium, the common price of the other firms' goods is higher than their Bertrand equilibrium price. For this result, it is not necessary that the quasi Bertrand reaction function of the other firms is always upward sloping. It is sufficient that the quasi Bertrand reaction function of the other firms lies more outward than their Bertrand reaction function, and the latter is upward sloping.

From Eq. (2) for Firm k, since $(\partial h_k(\boldsymbol{p})/\partial p_i) > 0$, we have

$$\frac{\partial \pi_k}{\partial p_i} = [p_k - c'(h_k(\boldsymbol{p}))]\frac{\partial h_k(\boldsymbol{p})}{\partial p_i} > 0, \quad \text{for } i \neq k.$$

Thus, the profit of Firm k increases along its Bertrand reaction function as the price of each other firm's good increases. Therefore, the profit of Firm k in the quasi Bertrand equilibrium is larger than its profit in the Bertrand equilibrium. □

Appendix B

Proof of Theorem 2. The quasi Cournot reaction function of the other firms lies more outward than their Cournot reaction function. The Cournot reaction function of Firm k

and that of the other firms are downward sloping, and the magnitude of the slope of these Cournot reaction functions is smaller than one. In the quasi Cournot equilibrium, the output of Firm k's good is determined along its Cournot reaction function, on the other hand, the common output of the goods of the other firms is determined along their quasi Cournot reaction function. Thus, in the quasi Cournot equilibrium, the common output of the other firms' goods is larger than their Cournot equilibrium output. For this result it is not necessary that the quasi Coumot reaction function of the other firms is always downward sloping. It is sufficient that the quasi Cournot reaction function of the other firms lies more outward than their Cournot reaction function, and the latter is downward sloping.

From Eq. (8) for Firm k, since $(\partial f_k(\boldsymbol{p})/\partial q_i) < 0$, we have

$$\frac{\partial \pi_k}{\partial p_i} = \frac{\partial f_k(\boldsymbol{p})}{\partial q_i} q_k < 0, \quad \text{for } i \neq k.$$

Thus, the profit of Firm k decreases along its Cournot reaction function as the output of each other firm's good increases. Therefore, the profit of Firm k in the Cournot equilibrium is larger than its profit in the quasi Cournot equilibrium.   □

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